Prescribed title number 9

Mathematicians have the concept of rigorous proof, which leads to knowing something with complete certainty. Consider the extent to which complete certainty is achievable in mathematics and at least one other area of knowledge.

Johanna Syrjänen
0571 021
Helsingin Suomalainen Yhteiskoulu
Theory of Knowledge Essay
May 2008
Word count: 1599

May 08
9. Mathematicians have the concept of rigorous proof, which leads to knowing something with complete certainty. Consider the extent to which complete certainty might be achievable in mathematics and at least one other area of knowledge.
Johanna Syrjänen, Helsingin Suomalainen Yhteiskoulu

Claiming to know something with full, absolute and perfect certainty is a very bold claim, to say the least. It is a claim that not all areas of knowledge can even think of making.[1] The title suggests that by the virtue of having the concept of rigorous proof, mathematical knowledge can achieve full certainty. By the extent of certainty, I understand it as the degree and character of certainty possible; degree in the sense of where a specific area of knowledge stands on a scale ranging from the extremes of complete certainty to complete uncertainty. Using mathematics and the natural sciences, traditionally held to be the subjects whose knowledge must pass rigid justification by reason and whose secure truths stand unaffected by passing fads and fickle human trends, as examples, I hope to show that the differences in character between mathematics and the natural sciences inevitably lead to differences in the degree and the nature, which are measures of the extent of certainty, achievable in each.

Mathematics is often expressed as the field of rigorous proof; proof is what defines the nature of mathematics. Deductive reasoning forms the basis of a proof; provided that the deductive argument holds, true conclusions will follow from true premises. Provided something is necessarily true, it must be known with complete certainty,[2] also. So, premises, or axioms, are the starting point in mathematics. Mathematics is an axiomatic system; all mathematical realms depend on the validity of the axioms they are built upon and hence the axioms are what certainty in mathematics relies upon. Given true axioms, complete certainty is fully achievable in mathematics. Axioms, by definition, can not be proved; infinite regress would follow and so mathematics must start from some statements- agreed to be axioms. So, how does one know that the axioms of, for example, Euclidean geometry, are true? The thing here is that one does not question whether Euclid's five postulates are true or

---

[1] Admittedly, the claim of certainty is something that some areas of knowledge do not need to and would not want to aspire to; for them it is irrelevant. For example, it is not possible for me to know with complete certainty, or indeed with any certainty that da Vinci's oil painting "The Last Supper" will inspire the same emotions in every viewer. Not only that, but the nature of the arts is such that different viewers may appreciate "The Last Supper", even though it invokes various emotional responses.

[2] Clearly, claims of complete certainty will only apply to the area of knowledge making that claim. If complete certainty existed in mathematics, I could not apply this certainty to any other area of knowledge; I would have to evaluate that area of knowledge on its on merit; certainty in mathematics, for example, would not imply certainty in religion.

1

Johanna Syrjänen, Helsingin Suomalainen Yhteiskoulu

not, but rather, asks if they are consistent. This is analogous to the rules of a game of chess- one does not ask whether the rules are "true", but one does expect to have a consistent set of rules, because from a consistent set of rules follows a playable game.[3] The same with mathematics; if the rules are consistent then one has a working mathematical system. This realization led to Riemannian geometry, which is based on axioms that are false in Euclidean geometry, but appear consistent in their own system, separate from Euclidean geometry. It can be proved that the only way for non-Euclidean geometry to contradict itself is for Euclidean geometry to contradict itself.[4] Likewise, Euclidean geometry is consistent provided that arithmetic is consistent.

And arithmetic? The only thing missing now for absolute certainty in mathematics is the proof that a system, for example arithmetic, actually is consistent. Enter Gödel, who gave mathematics Gödel's Incompleteness Theorem, and in doing so effectively sounded the death knell for absolute, beyond-a-speck-of-doubt certainty in mathematics. For thousands of years mathematicians have found no contradictions in Euclidian geometry. This does not mean that contradictions exist. It also does not mean that any do not, either. What it means is that the concept of rigorous proof can not be used to say whether or not they do. Gödel ultimately showed that if a system is consistent, there exist theorems that can neither be proved nor disproved, and there is no procedure that will prove a system consistent.[5] Hence complete certainty can not exist in mathematics.

Mathematicians may have the concept of rigorous proof, but the natural sciences have the concept of experiment. The method by which these areas of knowledge are done is noticeably different; the absolute rationalism of mathematics starkly confronts the empiricism of the natural sciences.[6] Unlike in mathematics, where we can create our own system by deciding the axioms through which to govern the system by, the idea of the natural sciences is to discover laws, not set them. One might argue that a mathematician is likewise discovering a world- not inventing it, but

[3] Ian Stewart, *From Here to Infinity* (NewYork: Oxford University Press, 1996) 54.
[4] Ian Stewart, *From Here to Infinity* (NewYork: Oxford University Press, 1996) 56.
[5] Ian Stewart, *From Here to Infinity* (NewYork: Oxford University Press,1996) 266-267.
[6] Notice though, that science does incorporate rationalism (the role it plays is significant and adds to the power of science), but empiricism is essentially missing as justification in mathematics.

in any case that world he "discovers" would be an exact mathematical one, instead of a rougher, cruder interpretation of it.

Because knowledge in the natural sciences is justified by experiment, the very nature of the natural sciences forces us to live with uncertainty, which creeps in from two separate quarters. Firstly, all experiments have an uncertainty associated with them, simply because the apparatus used will have an uncertainty associated with it. In measuring concentrations of phosphorus solution for my extended essay in chemistry, I used a spectrophotometer with an uncertainty of ± 0.01 mg/dm^3. So, in measuring a quantity of 30.00, I might actually have 30.01 or 29.99 mg/dm^3 of phosphorus. With these figures, the uncertainty is negligible, but supposing I get a reading of 0.00 milligrams phosphorus? The uncertainty of the apparatus will not let me know if the sample is indeed completely free of phosphorus, or whether there is still some phosphorus left. With more accurate instruments, we may decrease this uncertainty until its effect is again negligible, but it will always be present to some degree and can never be totally removed.

Secondly, scientific knowledge has an uncertainty associated with it because of the way it functions; the scientific method itself, the process of induction, prevents complete certainty from being achieved. Through scientific induction, one draws a general conclusion from a quantity of individual cases.[7] A high degree of certainty comes from induction, but there is a problem. We are making the leap from observed to unobserved, and this decreases the certainty in the natural sciences since even well-confirmed generalisations can let us down.[8] For example, all animals that suckle their young and give birth to live offspring were classified as mammals and animals that lay eggs are placed into categories separate from the class of Mamalia. This was well and good until the late 18th century, when platypuses were discovered in Australia.[9] They turned out to both suckle their young and lay eggs.

[7] Peter Mcinerny. *Introduction to Philosophy* (1992) 55.
[8] Before hastily criticizing the shortcomings of the scientific method, we should notice that Popper's falsification theory remedies this to some extent (a theory that will stubbornly not bend to falsification should be accepted as the best, but can not be held with complete certainty as it may be usurped by a better one, as in the case of Newtonian physics, or group selection in evolutionary biology), but ultimately shows that scientific theories cannot be conclusively verified or falsified, and thus complete certainty can not exist either.
[9] Bill Bryson, *In a Sunburned Country* (NewYork: Broadway Books, 2001) 274.

Regularities and laws seem especially present in the physical sciences, even when the quirks of biology let us down. The 18th century philosopher Hume suggested that if nature were uniform, we might be able to justify inductive generalisations.[10] However, Heisenberg, through his Uncertainty Principle, showed that nature is not uniform in the most fundamental cases by showing that one can never be exactly sure of both the position and velocity of a particle;[11] the more accurately one knows one, the less accurately one can know the other.[12] And so uncertainty lies at the very heart of the natural sciences and it is impossible to be completely rid of it.

The natural sciences and mathematics are both human endeavours. We are beings that view this world through our biased perceptions. Since sense perception plays a greater role in the natural sciences than in the abstractness of mathematics, because complete certainty is separated from mathematics only by Gödel's Incompleteness Theorem, and because there is nothing at all we can do about the extent of certainty achievable in mathematics, I venture to say that the extent of certainty achievable in mathematics is greater than that achievable in the natural sciences.[13] Not only is the degree of certainty different, but the nature of certainty is as well, which in turn affects the extent of certainty achievable: Individual mathematical theorems do not have a qualitative uncertainty associated with them. Mathematics is consistent or it is not; there is no middle ground. One could describe mathematics as a card house-- pull one card away and the whole structure tumbles down. Conversely, all scientific knowledge has an uncertainty associated with it, but this uncertainty can vary from one piece of scientific knowledge to the next. Scientific beliefs evolve over time, whereas mathematical proofs do not. For hundreds of years we believed Newton's laws of motion to be the most fundamental in physics, until Einstein expanded them. Einstein showed that Newton's laws work only on our local scale, but proposed special relativity, which applies in a more universal context. Newton's work is contained in Einstein's. Certainty in the natural sciences is

[10] Peter Mcinerny. *Introduction to Philosophy* (1992) 55.
[11] This limit comes from the nature of quantum mechanics, not through the ineptness of an experimenter (Tim Kirk, *Physics for the IB Diploma* (Oxford: Oxford University Press, 2003) 121)
[12] Stephen Hawking, *A Brief History of Time* (Bantam Books, 1988) 190.
[13] One could argue that the natural sciences reduce from biology to chemistry to physics, which in turn reduces to mathematics, but the matter stands that science is not only mathematics, but has a strong element of experiment in it, which is missing from mathematics.

May 08
9. Mathematicians have the concept of rigorous proof, which leads to knowing something with complete certainty. Consider the extent to which complete certainty might be achievable in mathematics and at least one other area of knowledge.
Johanna Syrjänen, Helsingin Suomalainen Yhteiskoulu

analogous to approaching an asymptote but, by definition, never quite reaching it. Empirical evidence is not proof; no amount of experimental evidence can prove knowledge. If I prove that the angles of a triangle add to 180º, I know something that universally applies to all triangles,[14] without investigating each triangle individually. However, as we saw with the platypus example, inductive logic leaves uncertainty in scientific knowledge.

It seems that all we can be completely certain of, coolly and impartially, is of not being fully certain. In a sense this is anticlimactic; after enjoying thousands of years of supposed certainty with the Greeks, Euler, Gauss and finally the 20th century, mathematics is not infallible after all. However, this need not necessarily be such a dreadful thing as it first seems; scientific knowledge is a collection of statements of varying degrees of certainty- but none absolutely certain; for science to progress there must be room for ignorance and doubt,[15] and this, the breathtaking possibility to explore the foundations and crevasses of our physical world and stand on the shoulders of giants while doing so, is what gives the natural sciences its awe-striking charm. And would we admire a person who justified their infallibility by "because I say so"? Likewise, how can a system demonstrate its own infallibility?[16] Complete certainty misses mathematical knowledge by a hair's breadth, but leaves mathematics with an element of elusive mystery and allure to add to the beauty and elegance of mathematical reasoning.

---

[14] Notice that we are in Euclidean geometry here!
[15] Richard Feynman, *The Pleasure of Finding Things Out* (Penguin Books, 2007) 146.
[16] Ian Stewart, *From Here to Infinity* (NewYork: Oxford University Press,1996) 267.

Bibliography:

Bryson, Bill. *In a Sunburned Country.* NewYork: Broadway Books, 2001.

Feynman, Richard. *The Pleasure of Finding Things Out*. Penguin Books, 2007

Hawking, Stephen. *A Brief History of Time*. Bantam Books, 1988.

Kirk, Tim. *Physics for the IB Diploma*. Oxford: Oxford University Press, 2003.

Mcinerny, Peter. *Introduction to Philosophy*, 1992.

Stewart, Ian. *From Here to Infinity*. NewYork: Oxford University Press,1996.

